C2 JUNE 2009

1 $\int_1^4 \left(2x + 3\sqrt{x}\right) dx$

$\int_1^4 2x + 3x^{1/2}\, dx$

$\left[x^2 + 2x^{3/2} \right]_1^4$

$= 32 - 3$

$= \underline{\underline{29}}$

2a) $(2+kx)^7$

$= 2^7 + \binom{7}{1} 2^6 \times kx + \binom{7}{2} 2^5 \times (kx)^2$

$= 128 + 448kx + 672k^2x^2$

b) coeff of x^2 = $6 \times$ coeff of x

$6 \times 448k = 672k^2$

$2688 = 672k$

$\underline{\underline{k = 4}}$

3a) $f(k) = (3k-2)(k-k) - 8$

$f(k) = -8$

b)

when $f(2)$ remainder $= 4$

$f(2) = (3\times2-2)(2-k) - 8$

$= 4(2-k) - 8$

$= 8 - 4k - 8$

$= -4k$

$-4k = 4$ $\quad \underline{\underline{k = -1}}$

c) $f(x) = (3x-2)(x+1) - 8$

$= 3x^2 + x - 10$

$f(x) = (3x-5)(x+2)$

$x^2 + x - 30$

$(x-5)(x+6)$

$(3x-5)(x+2)$

4a) $y = \sqrt{2^x + 1}$

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{2^x+1}$	1.414	1.554	1.732	1.957	2.236	2.580	3.000

b)

$0.5 \times \frac{1}{2}\left[(1.414 + 3) + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)\right]$

$= 6.133$

c) the approximation is an overstatement since the trapezia lie over the line.

5a) $ar^2 = 324$ $\qquad ar^5 = 96$

$a = \frac{324}{r^2}$ $\qquad a = \frac{96}{r^5}$

equate $\frac{324}{r^2} = \frac{96}{r^5}$

$324r^5 = 96r^2$

$324r^3 = 96$

$r^3 = \frac{8}{27}$

$r = \frac{2}{3}$

b) $a = \dfrac{324}{\left(\frac{2}{3}\right)^2} = \underline{\underline{729}}$

c) $S_{15} = \dfrac{729\left(1-\left(\frac{2}{3}\right)^{15}\right)}{1-\frac{2}{3}} = 2187\left(1-\left(\frac{2}{3}\right)^{15}\right)$

$= \underline{\underline{2182}}$ (4.s.f)

d) $S_\infty = \dfrac{a}{1-r} = \dfrac{729}{1/3} = \underline{\underline{2187}}$

6a) $x^2+y^2-6x+4y=12$

$(x-3)^2-(3)^2+(y+4)^2-\left(\frac{4}{2}\right)^2-12=0$

$(x-3)^2+(y+\frac{4}{2})^2-25=0$

$(x-3)^2+(y+2)^2=25$

centre $= \underline{(3,-2)}$ radius $\sqrt{25} = \underline{5}$

b)

length between points $= \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$\overset{x_1\ y_1}{P(-1,1)}$ $\overset{x_2\ y_2}{Q(7,-5)}$

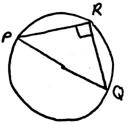
length $= \sqrt{(-5-1)^2+(7--1)^2} = \sqrt{(-6)^2+8^2}$

length $= 10$

$10 = 2\times 5 = 2\times$ radius

$\therefore$ PQ must be the diameter.

R must lie on circle



c) lies on positive y axis $\therefore x=0$

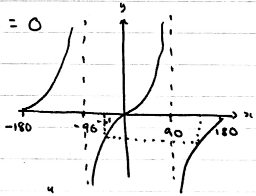
$$y^2+4y=12$$
$$y^2+4y-12=0$$
$$(y+6)(y-2)=0$$
$y=-6$ or $y=2$ positive y axis $\therefore \underline{\underline{y=2}}$

when $y=2$

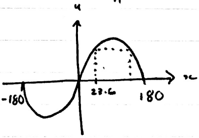
$$x^2+(2)^2-6x+4(2)=12$$
$$x^2-6x=12-8-4$$
$$x^2-6x=0$$
$\underline{x=6}$ $\underline{\underline{R(6,2)}}$

7 i) $-180 \le \theta < 180$

$$(1+\tan\theta)(5\sin\theta-2)=0$$



$\tan\theta=-1$
$\theta=-45$
$\theta=180-45$
$\underline{\theta=-45,\ 135}$



or $\sin\theta=\frac{2}{5}$

$\theta=23.6$
$\theta=180-23.6=156.4$
$\underline{\theta=23.6,\ 156.4}$

ii) $0 \leq x < 360$

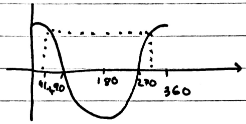
$4\sin x = 3\tan x$

$4\sin x = \frac{3\sin x}{\cos x}$

$4\cos x = 3$

$\cos x = \frac{3}{4}$

$x = \cos^{-1}\left(\frac{3}{4}\right) = 41.4$



$90 - 41.4$
$= 48.6$
$270 + 48.6$
$= 318.6$

$x = 41.4, \ 318.6$

8a)

$\log_2 y = -3$

$2^{-3} = y \Rightarrow y = \frac{1}{8}$

b) $\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$

$\frac{9}{\log_2 x} = \log_2 x$

$9 = (\log_2 x)^2$

$\pm\sqrt{9} = \log_2 x$

$\pm 3 = \log_2 x$

$2^3 = x$ or $2^{-3} = x$

$8 = x$ $\qquad x = \frac{1}{8}$

9 surface area

a) area of sector $= \frac{1}{2}r^2\theta \quad = \frac{1}{2}r^2$

$2 \times \frac{1}{2}r^2 + 2h \times r + rh$

$r^2 + 2hr + hr$

$r^2 + 3hr$ = surface area

volume:

$\frac{1}{2}r^2 \times h = 300$

$r^2 h = 600$

$h = \frac{600}{r^2}$

Surface area $= r^2 + 3\left(\frac{600}{r^2}\right)r$

$= r^2 + \frac{1800}{r}$ cm^2

b) $S = r^2 + 1800r^{-1}$

$\frac{ds}{dr} = 2r - \frac{1800}{r^2} = 0$

$2r^3 - 1800 = 0$

$2r^3 = 1800$

$r^3 = 900$

$r = 9.7$

c) $\frac{d^2s}{dr^2} = 2 + \frac{3600}{r^3}$ when $r = 9.7$

$= 2 + \frac{3600}{(9.7)^3} \approx 5.9$

$\frac{d^2s}{dr^2} > 0$ $\therefore$ it is a minimum

d) ~~min value = 55.6 cm²~~

$S_{min} = (9.65)^2 + \frac{1800}{9.65} = 279.65\ldots$

$= \underline{\underline{280}}$